FORECASTING THE OCCURRENCE OF MONTEREY PRECIPITATION FROM 500-mb DATA

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Lieutenant Junior Grade, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
AEROLOGY

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

An attempt was made to forecast the occurrence of Monterey precipitation using only 500-mb data. The use of the wind speed and direction and the 24-hour wind direction change at weather ships Papa and November gave no significant results for the period of December 1955 to March 1956.

Using West Coast stations, a probability forecasting graph was plotted from the height difference between Santa Maria and Oakland and the 500-mb temperature at Oakland. The results on an independent sample of 151 cases resulted in 75% correct forecasts with a skill score of 0.29.

With a sample of 72 the 2-parameter forecast was 68% correct with a skill score of -0.08. When a third parameter, 500-mb wind direction at Oakland, was added, the accuracy increased to 73% correct with a skill score of 0.11. For this set of data neither method gave significant results.

Although these results do not show the accuracy that other objective methods generally have, these results show the possibility of forecasting surface precipitation from 500-mb data.

The study of the winter time precipitation at Monterey was undertaken in the spring of 1957 at the U. S. Naval Postgraduate School with the purpose of forecasting the occurrence of Monterey precipitation from 500-mb data. The author wishes to express his appreciation to Professor W. D. Duthie for his valuable guidance and heartening encouragement. The author is also grateful to the Engineering Department of Del Monte Properties Company for free access to their weather observational data.

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TABLE OF SYMBOLS

Symbol	Meaning
1. dd	Wind Direction to a 36-point Compass at 500 mbs
2. ff	Wind Speed in knots at 500 mbs
3. $\Delta H_{SMX-OAK}$	The 500-mb height at Santa Maria minus that at Oakland
4. $\Delta H_{OAK-MFR}$	The 500-mb height at Oakland minus that at Medford
5. TT_{OAK}	The 500-mb temperature at Oakland

1. Introduction

Objective forecasts of two types are presently used to produce faster, more accurate forecasts and to supplement the technique of forecasters who are less experienced in the idiosyncracies of specific localities. The user of Vernon's method [1] for predicting precipitation at San Francisco objectively types weather situations and then uses surface information to find the forecast of "rain" or "no rain." Brier, in his forecast of precipitation for Washington, D. C. [2] and the TVA Basin [3], developed a method for forecasting the probability of precipitation using as many parameters as are necessary and pertinent. On the West Coast, Jorgenson [4] applied this method to forecasting the occurrence of rain in the San Joaquin Valley of central California, and Thompson [5] similarily quantitatively forecast precipitation for Los Angeles.

The probability forecast has a definite advantage over the categorical forecast. To be useful the categorical forecast must be better than, or nearly as good as the subjective forecast. Although an experienced forecaster may have a feeling of how accurate his subjective forecast is, few if any can or have ever tried to express their confidence in the form of a percentage. On the other hand even a probability forecasting method which does not quite measure up to subjective forecasts in the categorical field may accurately indicate the probability of rain in every instance. The categorical forecast may be thought of as having the 50% frequency line as the divider between "rain" and "no rain." This 50% line may not always be the deciding line for attempting an operation. One must decide in each case by the cost of the operation in money or human life, the gain to be expected, and the loss to be expected from

the failure of the weather element. The maximum or minimum probability of rain for the operation under consideration may be calculated. This process of acting on calculated risks, though universally used for a long time in other economic practices, had to wait for probability forecasting to be applied to the weather. The U. S. Weather Bureau has found their experiment in issuing probability forecasts successful. Even the Sunday afternoon picnicker wants to know how sure it is to be a clear day.

Since the Numberical Prediction Center in Washington, D. C. produces a much better 500 than 1000-mb prognostic chart, the present development of a probability forecasting technique for the occurrence of rain in Monterey used only 500-mb data. The method itself was developed using actual radiosonde data, but it was expected that the forecast might be extended using the numerical prognosis at 500 mbs. This present study is unique in that all other methods in the literature depend to a certain extent on surface data. Even Jorgensen [4] used surface data to type weather situations before forecasting with 700-mb parameters. The aforementioned methods relied solely on the 700-mb level for their upper-air data. Now that there is a sufficient history of accurate 500-mb data, reliance on this lower level for all upper-air data is not necessary.

The use of the raw data from radiosonde reports has the advantage of being more objective than the subjectively analyzed data. The use of teletype data also allows the forecaster to solve his problem a couple of hours sooner than if he needed an analyzed chart. The disadvantages of using this raw data are the possibility of unrepresentative data and missing or garbled reports. In spite of the fact that tho data at 500 mbs is much more representative of the area than that below, and that this

data is recorded and transmitted much more reliably than before, forecasts on a "perfect 500-mb prognosis" probably would be as good as those using actual data.

To verify Vernon's hypothesis [1] that his method could be used satisfactorily without modification within a 150 mile radius of San Francisco, MacLeod [7] and Jepson [8] tested this method at Monterey. Although the results agreed quite well with Vernon's results [1], one should not assume that the pattern for Monterey precipitation is greatly similar to that of San Francisco. Even on the Monterey Peninsula there may be considerable variation. The precipitation measurements for this research were taken at Pebble Beach. In general Monterey precipitation occurs at the same time in somewhat larger amounts. The fact that this method was specifically developed for Pebble Beach precipitation should not hinder its use anywhere on the peninsula, especially in view of the studies of MacLeod [7] and Jepson [8].

2. Preliminary Studies on the 1955--56 Rainy Season

In this initial work precipitation measurements from the Naval Air Facility for December 1955 through March 1956 were used. In this period of 122 days 46 rainy days were recorded.

In order to test the local forecasting rule that a SW wind in Monterey is followed by rain the next day, the concurrence of these two phenomena was tabulated. Although precipitation and SW winds individually occurred 46 times, they coincided in the proper manner on only 14 occasions or 30% of the time. During this particular year the rule had no significance, for rain itself averaged a 39% chance of occurrence. This short sidetrack was the only deviation from the use of 500-mb data as predictors.

Attempts were made to correlate the 500-mb radiosonde data from weather ships "Papa" and "November" with rain at the Naval Air Facility. The first test involved the correlation of a 24-hour back or veer of the wind and subsequent precipitation in Monterey. Table 1 shows the results of this experiment. The total number of cases for each column is less than

No of days later rain occurred	Papa				November			
	Back		Veer		Back		Veer	
0	16	55%	4	31%	13	39%	8	42%
1	9	31	4	46	16	48	5	26
2	12	41	7	54	15	45	7	37
3	13	45	7	54	13	39	5	26
4	11	38	8	62	14	42	6	32
5	11	38	8	62	17	51	4	21
Total No. Cases	29		13		33		19	

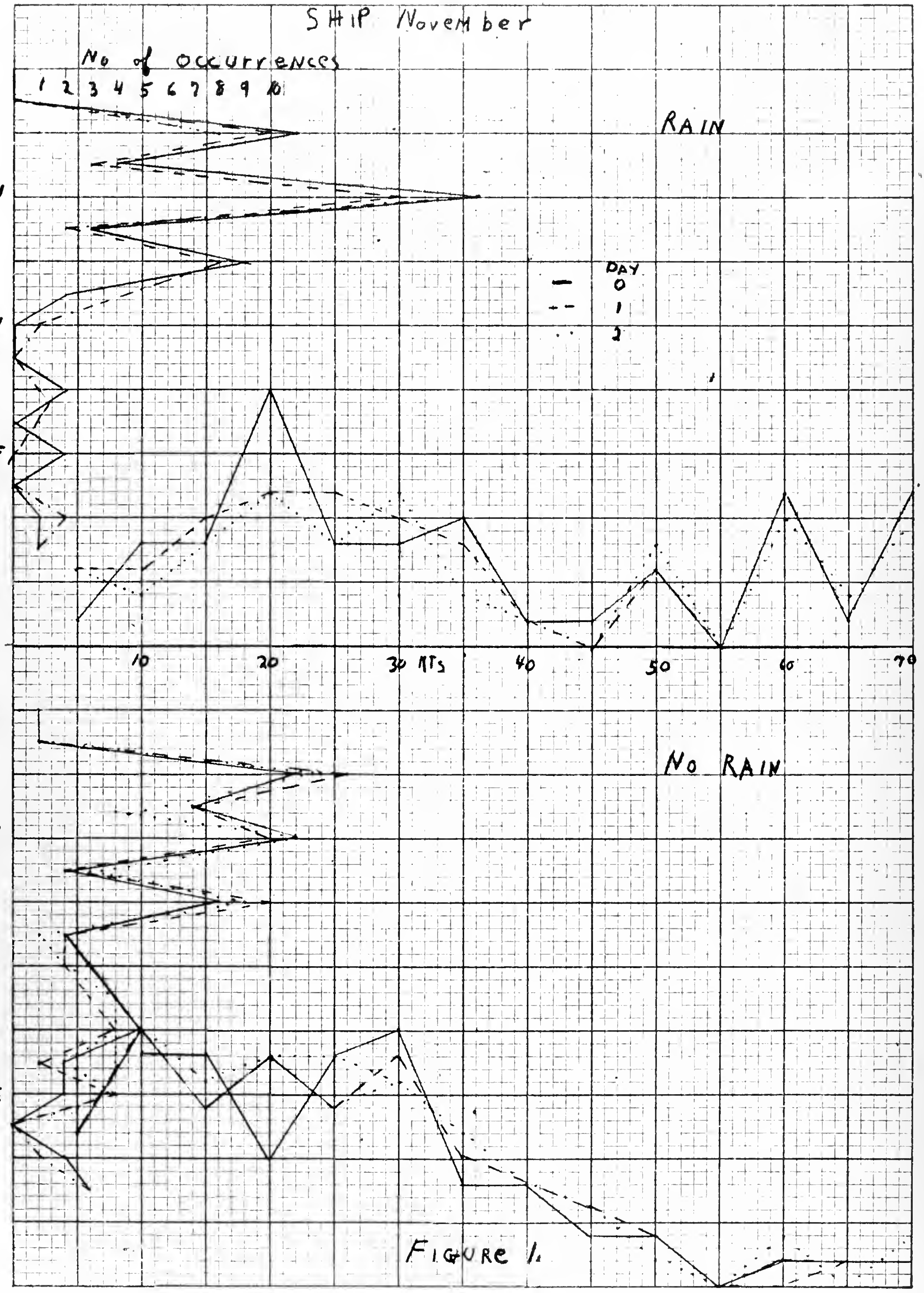
Table 1

the sum of each column because each variation in wind direction was often the forerunner of several rainy days. Monterey precipitation generally persists for a few days. The verification percentages are, for each type, the specified number of days after the indicated change of wind direction.

Even from this small sample it can be seen that this change does not in itself provide a significant forecasting tool. Actually the best results were found for four to five days after veering happened. Even though weather ship Papa is a considerable distance upstream from Monterey, it is hardly conceivable that events happening four or five days previously could be more important than more recent events. Actually on the basis of the size of the sample there is little significant difference between each day. There does seem to be some significance in the fact that the verification percentages for Papa increase for veering winds and decrease for backing winds as the length of the forecast period increases. The observations for weather station November seem to be more random with less differentiation between veering and backing.

An attempt was made to show the effects of wind direction and speed at Papa and November and subsequent rain in Monterey. Graphs for the occurrence and non-occurrence of rain after zero, one and two days were plotted for November and Papa. Figures 1 and 2 show these plots. The length of the forecast period showed no significance. Using the mean values for these days, Figure 3 was plotted to show the difference between days of rain and no rain. Here again the two plots did not differ significantly. Although the frequencies differed, the maxima and minima for the rain and no-rain cases came at the same wind speeds or directions.

From these preliminary studies it was concluded that although reports from Papa and November might be connected with subsequent rain in Monterey, more usable information might be obtained using stations closer to Monterey.



SHIP November
No of occurrences
1 2 3 4 5 6 7 8 9 10
S
W
N
E
RAIN
DAY
0
1
2
10
20
30 KTs
40
50
60
70
S
W
N
E
No RAIN

FIGURE 1.

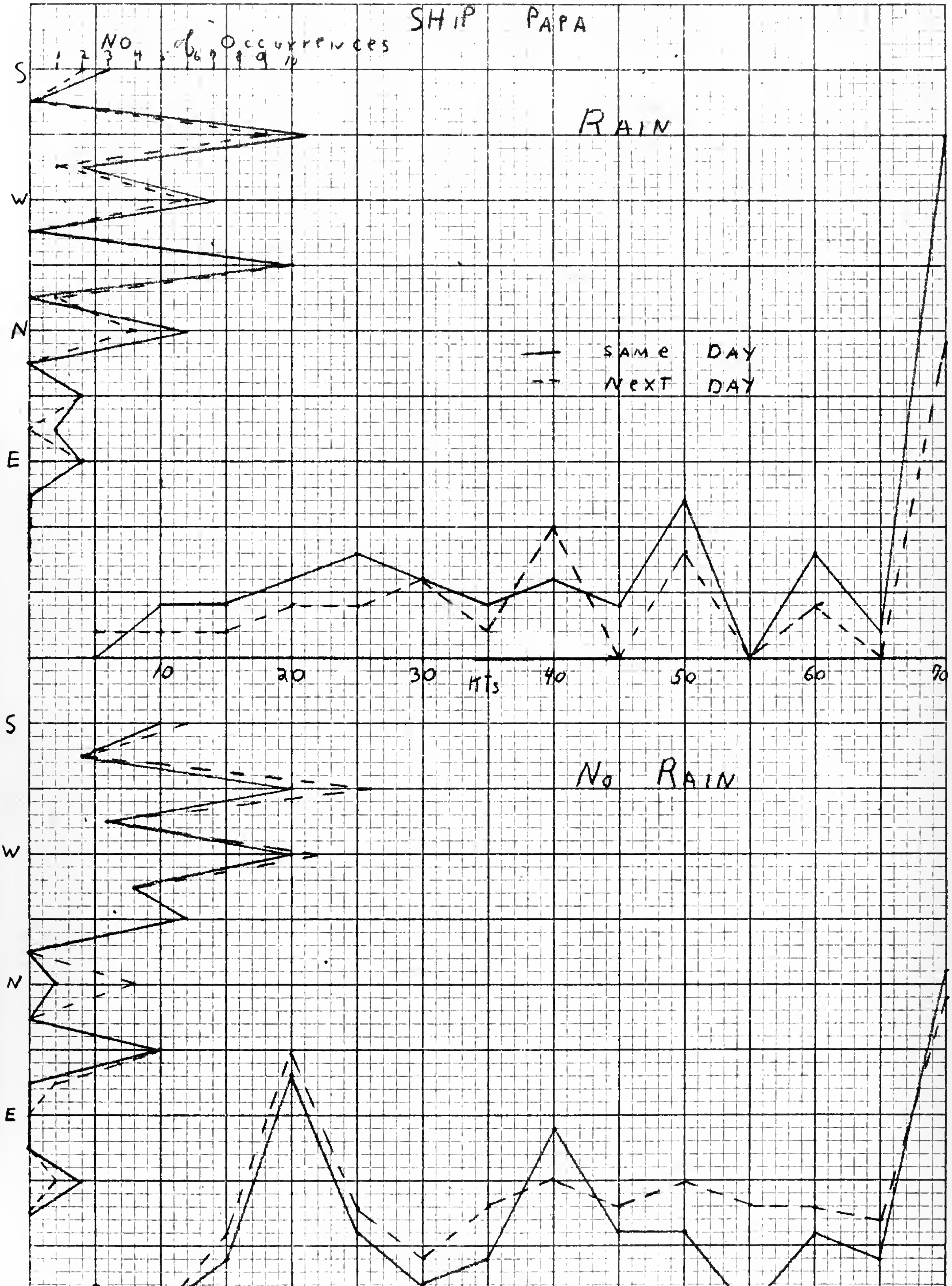

SHIP PAPA
NO. of Occurrences
1 2 3 4 5 6 7 8 9 10
S
W
N
E
RAIN
SAME DAY
NEXT DAY
10
20
30
KTs
40
50
60
70
S
W
N
E
No RAIN

FIGURE 2.

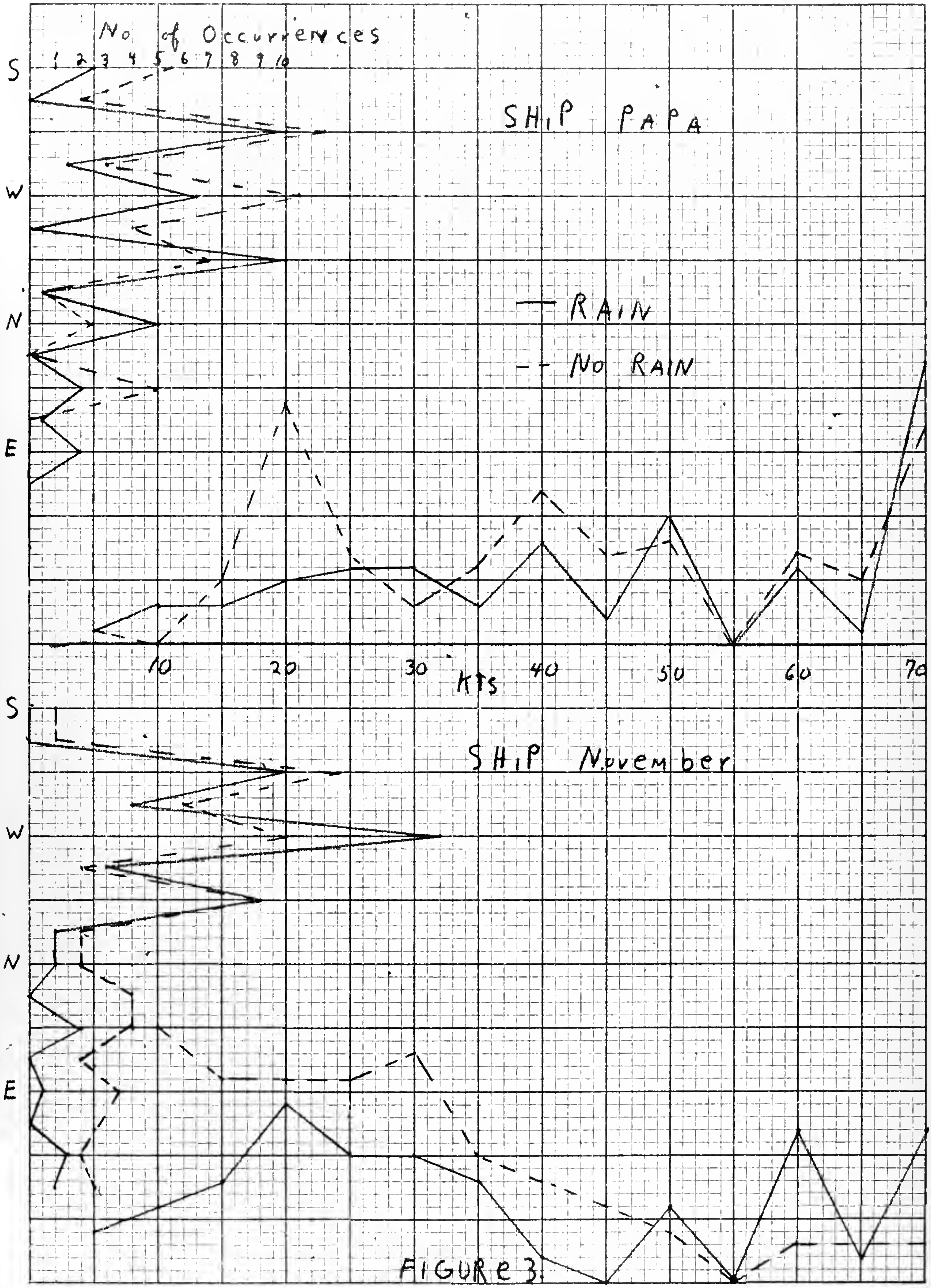

No of Occurrences
1 2 3 4 5 6 7 8 9 10
S
W
N
E
SHIP PAPA
RAIN
No RAIN
10
20
30
40
50
60
70
KTS
SHIP November

FIGURE 3

3. Use of West Coast Reports

Since the failure to find any correlation between the 500-mb reports of the weather ships and the occurrence of Monterey precipitation could be blamed to a large measure on the distance between these stations and Monterey, the next effort was devoted to the correlation of West Coast 500-mb reports and Monterey rain. Table 2 lists the twelve winter months

1949	January	February	March	December
1950	January	February	March	
1951			March	December
1952	January	February	March	

Table 2

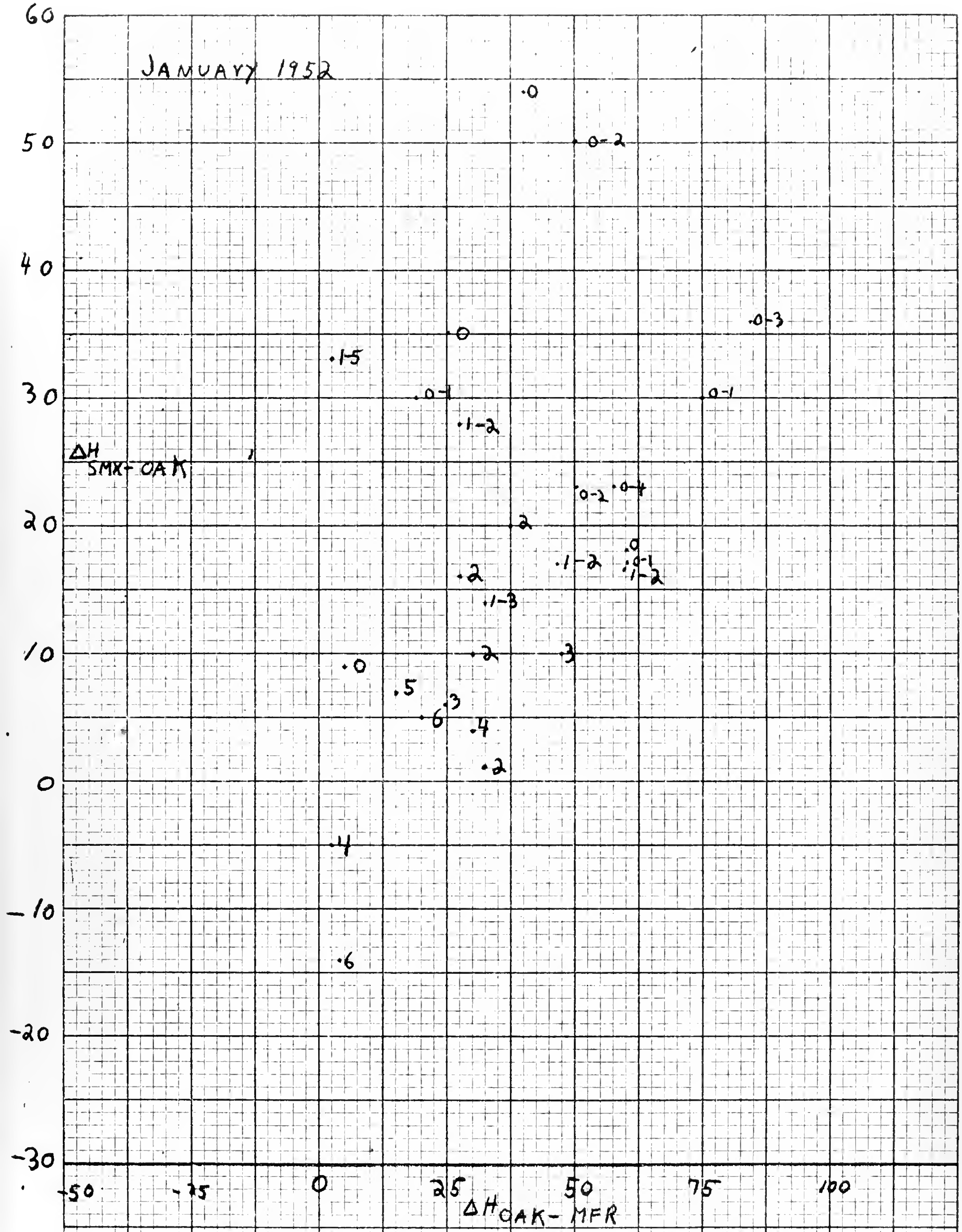
which were used for this study. All upper-air data were taken from the Synoptic Series [6]. The precipitation reports taken by the Del Monte Properties Company at Pebble Beach were used because they were the best available source of data. The available California Summaries were incomplete. Rain measurements from the Naval Air Facility differed slightly from those taken at Pebble Beach; therefore all the results of this study should only apply to Pebble Beach.

In the Pebble Beach data there were some days in which no record was entered. These days, along with the days in which the pertinent information was missing, were not used. In spite of these omissions, approximately 330 cases were used for each study. Brier [2] used 271 cases for precipitation studies in the 3-month period from December through February. Thompson [5] employed approximately 360 cases for his precipitation study covering the six months from October through March. On the basis of these past studies, the present sample seemed adequate.

Three stations were chosen so that the lateral variation of the parameters could be measured. Since nearly all storm activity comes

from northwest of Monterey, Santa Maria and Oakland were chosen to bracket Monterey, and Medford was selected to indicate the approach of weather-producing phenomena from the north. When a single station was used, the Oakland reports were naturally used because this station is both near and north of Monterey.

The 500-mb parameters recorded for each station were the height of this surface, the temperature and the wind direction and speed. Preliminary scatter diagrams using $\Delta H_{SMX-OAK}$ vs $\Delta H_{OAK-MFR}$ and the plot of $\Delta H_{SMX-OAK}$ against TT_{OAK} were used to decide the most favorable length for the forecast. Beside each point the number of days until precipitation occurred was plotted. When precipitation persisted for several days, the number of days till the termination of precipitation was also plotted since the parameters might be construed as forecasting rain for any of these days. Figure 4 is an example of one of these plots. From these scatter diagrams the best forecast was determined to be for the same or the following day. The radiosonde reports published in the Synoptic Series [6] are recorded for 0300Z, which is 1900 PST of the previous day. The Pebble Beach rain gauge was read daily at 0815 PST, and therefore the length of time until the beginning of the forecast period was $13\frac{1}{4}$ hours. At the time of receipt of teletype data this forecast would be for approximately ten hours. Thompson [5] estimated a six-hour forecast after map time. Brier [1] forecast for a 24-hour period starting six hours after a local surface observation. Thus if significant results can be obtained in this study a considerably longer forecast period may be used, even in cases in which missing data requires the partial analysis of the 500-mb chart. In order to overreach the 24-hour prognosis of



JANUARY 1952
ΔH SMX-OAK
ΔH OAK-MFR
60
50
40
30
20
10
0
-10
-20
-30
-50
-25
0
25
50
75
100

FIGURE 4.

Vernon [1] the use of a good prognostic 500-mb chart would be necessary.

In the graphical integration technique suggested by Brier [2], any number of variables may be combined into one forecast. Two parameters are plotted against one another, and each point is marked appropriately for the occurrence or non-occurrence of the meteorological event. This scatter diagram is then analyzed into isopleths of percentage occurrence. The values of these isopleths are used as a new variable derived from the original pair in the construction of a new scatter diagram. In this step-wise manner any number of variables may be combined into one forecast. At each stage the derived variable should represent a more definitive forecast. The procedure seems very simple, but the accuracy of the forecast depends on the selection of the most significant variables and the proper analysis of the scatter diagrams.

In the present application of Brier's technique [3] variables were selected on the basis of their availability as well as their forecast significance, so that a reasonably good forecast could be made from teletype data with partially missing reports. Dew-point temperatures were omitted because they were so often missing. Since wind direction and speed were frequently absent, they were omitted from the "first approximation" to the forecast. Only the heights and the temperatures at the 500-mb level were used initially.

4. The Correlation of Monterey Precipitation with $\Delta H_{SMX-OAK}$ and $\Delta H_{OAK-MFR}$

Figure 5 is the scatter diagram of $\Delta H_{SMX-OAK}$ plotted against $\Delta H_{OAK-MFR}$. Crosses indicate no subsequent precipitation at Pebble Beach, and numbered circles indicate the amount of rainfall. This diagram was then divided into areas having ten or more observations. The fractional and percentage occurrences of rain were indicated in each area. A X^2 test [9] was run to show that the distribution of points was significantly different from chance. The formula for evaluating X^2 is

$$X^2 = \sum_{i}^{k} \frac{(o_i - e_i)^2}{e_i}$$

where "o" is the observed frequency and "e" is the chance frequency. The level of significance of this sum is then found from a table of X^2 values tabulated against the number of degrees of freedom. Because the X^2 test is only meaningful for values of e greater than five, adjacent blocks having similar composition were combined as illustrated in Table 3. The results of this test show that the distribution of points within the cells differs significantly from chance well beyond the 0.001 confidence level. This agreed with the results of Brier [2].

The X^2 test is a necessary but not sufficient condition for the construction of a proper percentage forecasting chart. In this test no account can be taken of the juxtaposition of blocks of widely differing percentages of occurrence. In this instance the isolines could not be plausibly drawn, and therefore the diagram had to be abandoned.

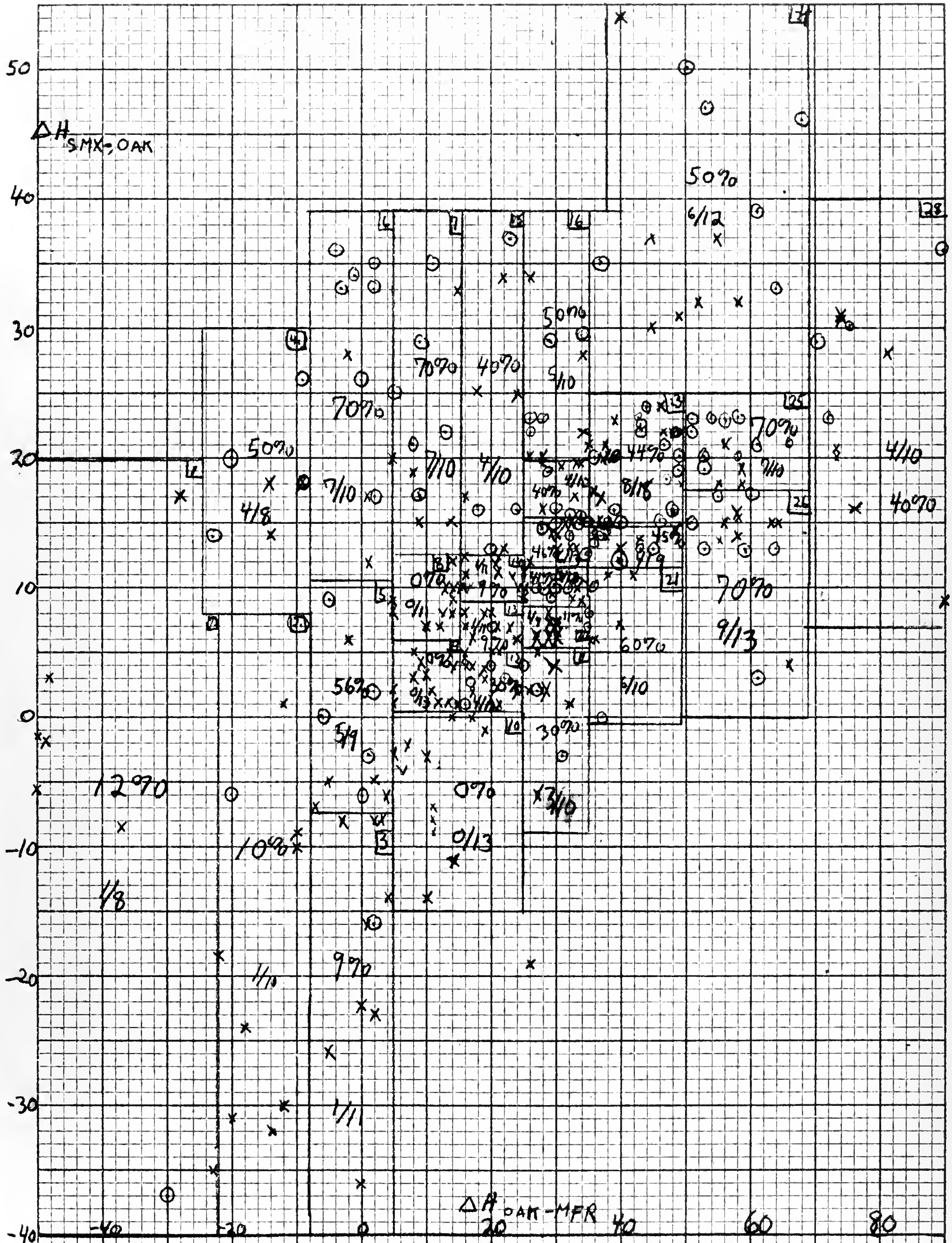

ΔH SMX-OAK
ΔH OAK-MFR
50
40
30
20
10
0
-10
-20
-30
-40
-40
-20
0
20
40
60
80
50%
6/12
70%
70%
40%
5/10
50%
7/10
7/10
4/10
4/8
44%
8/18
40%
70%
4/10
40%
70%
9/13
60%
6/10
56%
5/9
30%
0%
3/10
0/13
12%
10%
1/8
1/10
9%
1/11

FIGURE 5.

Block Nos.	No. of Cases	Observed	Expected	$(o-e)^2/e$
1, 2, & 3	29	3	10.4	5.26
4 & 5	17	9	6.1	1.38
6 & 7	20	14	7.2	6.42
8, 9, & 10	37	0	13.3	13.30
11 & 12	22	7	7.9	0.12
13, 14 & 20	32	3	11.5	6.28
15 & 16	20	9	7.2	0.45
17, 18 & 19	36	15	12.6	0.46
22 & 23	27	12	9.7	0.55
25 & 26.	26	18	9.3	8.14
24, 25 & 28	33	17	11.8	2.29
				44.66

10 degrees of freedom X^2 = 29.6 at the 0.001 level

Table 3

5. Correlation of Monterey Precipitation with $\Delta H_{SMX-OAK}$ and TT_{OAK}

From a marginal analysis of Figure 5 it was determined that the occurrence of Monterey precipitation was more highly correlated with $\Delta H_{SMX-OAK}$ than with $\Delta H_{OAK-MFR}$. Therefore the former variable was retained and plotted against TT_{OAK}. This plot had the advantage of requiring data only from two stations instead of three. In the division of the scatter diagram into blocks an attempt was made to make the blocks as nearly square as possible. In this manner the percentage of occurrence for each block can be said to represent that of the center with the least error. Figures 6 and 7 show the scatter diagram and the block diagram, respectively. Although the X^2 test of the data, as shown in Table 4, did not show significance quite to the 0.001 level, analysis into isopleths of percentage as illustrated in Figure 8 was easily accomplished.

Block Nos.	No. of Cases	Observed	Expected	$(o-e)^2/e$
1 & 2	23	13	8.3	2.66
3, 7 & 8	36	18	13.0	1.92
4	15	11	5.4	5.69
9 & 12	22	11	7.9	1.21
5, 13 & 14	35	19	12.6	3.26
10 & 11	27	9	9.7	0.05
6 & 15	28	5	10.1	2.58
16	17	4	6.1	0.72
12 & 17	21	7	7.6	0.04
20, 22 & 23	27	10	9.7	.01
18	15	0	5.4	5.41
19	18	3	6.4	.81
21, 24 & 25	33	4	11.9	5.24
				30.60

12 degrees of freedom $X^2 = 32.4$ at the 0.001 level

Table 4

This prediction graph was tested against data for January, February and March of 1953. Since the data from Pebble Beach were not recorded on Sundays in this year, no forecasts for Sunday and Monday were made when

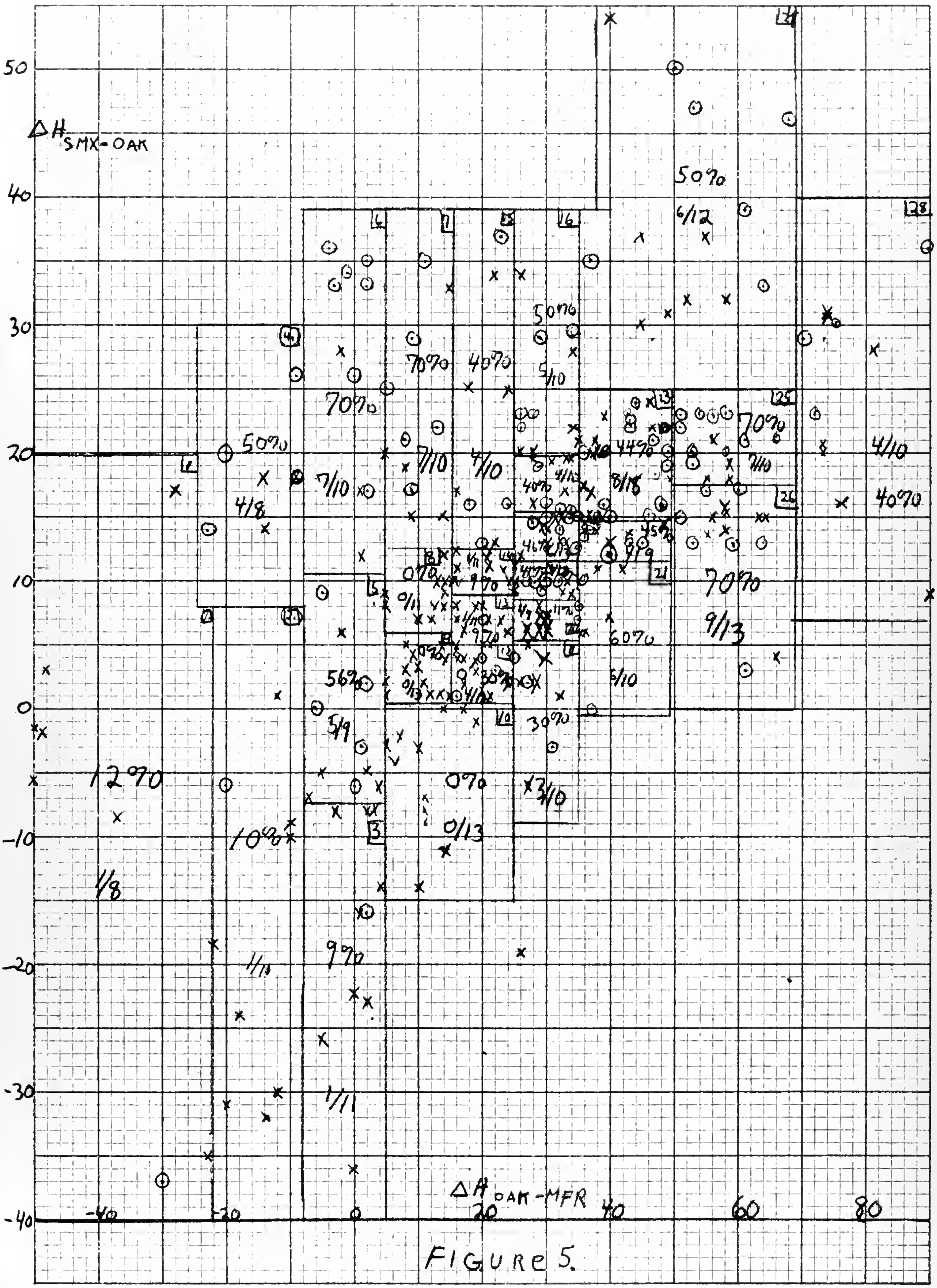


FIGURE 5.

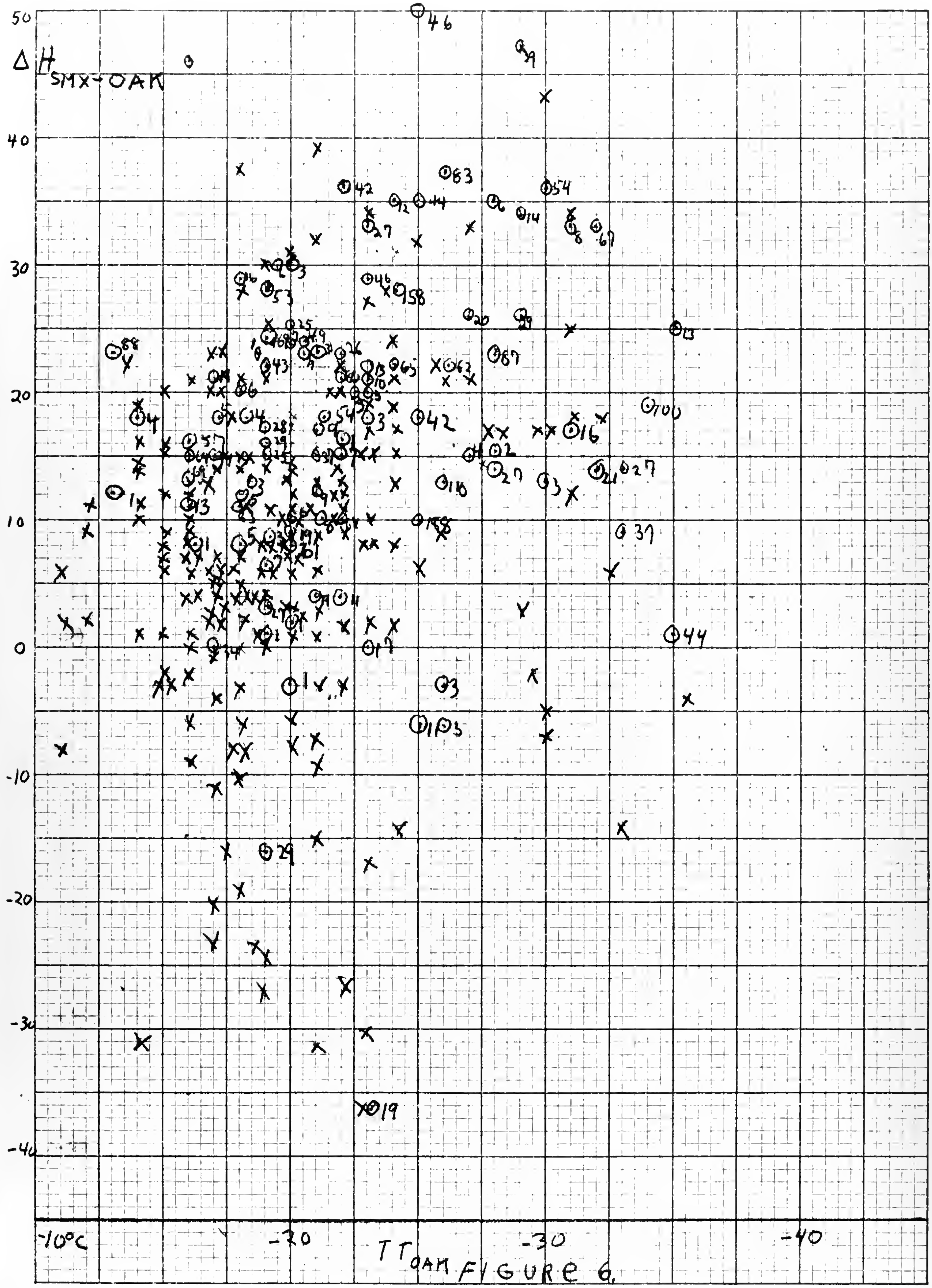
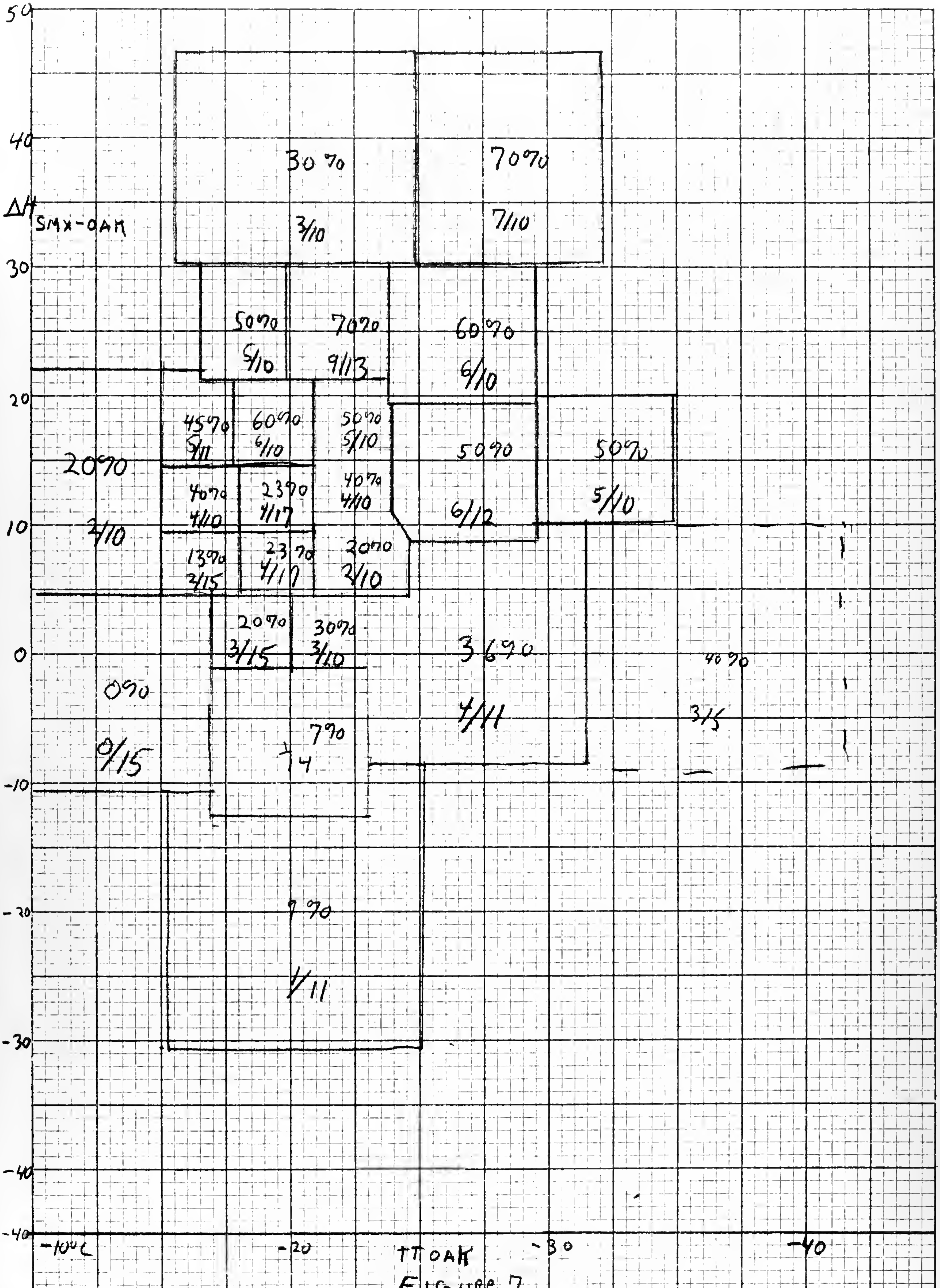


FIGURE 6.


50
40
30
20
10
0
-10
-20
-30
-40
-40
ΔH SMX-OAK
30%
3/10
70%
7/10
50%
5/10
70%
9/13
60%
6/10
45%
5/11
60%
6/10
50%
5/10
50%
6/12
50%
5/10
20%
2/10
40%
4/10
23%
4/17
40%
4/10
13%
2/15
23%
4/17
20%
2/10
20%
3/15
30%
3/10
36%
4/11
40%
3/5
0%
0/15
7%
1/14
7%
1/11
-10°C
-20
TT OAK
-30
-40

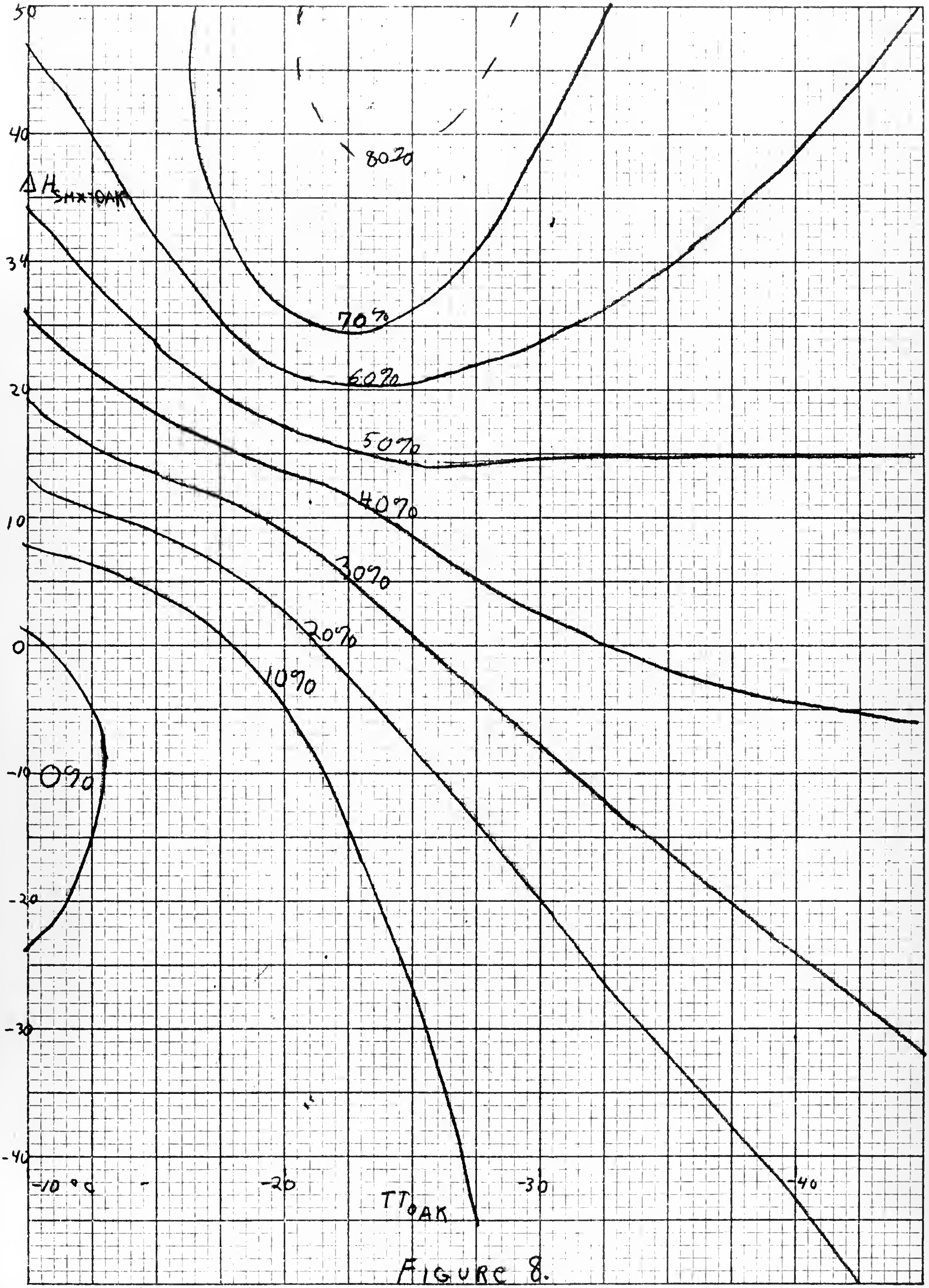

50
40
34
20
10
0
-10
-20
-30
-40
ΔH SMX-OAK
80%
70%
60%
50%
40%
30%
20%
10%
0%
-10 °C
-20
-30
-40
TT OAK

FIGURE 8.

the reading for Monday showed a two-day accumulation. Since this sample of 79 was too small to test the breakdown of each ten percentage line, this data was only tested on a categorical forecast using 50% and greater as the area of the graph for forecasting rain. Table 5 shows that the forecast verified 81% with a skill score of 0.45.

		Forecast		
		Rain	No Rain	Total
	Rain	8	7	15
Observed	No Rain	7	57	64
	Total	15	64	79

Table 5

The skill score is defined as:

$$S_c = \frac{C - E_c}{T - E_c}$$

where C = the number of correct forecasts,

E_c = the number of forecasts expected to be correct due to chance,

T = the total number of forecasts.

S_c has a value of unity when all forecasts are correct, and a value of zero when the number of correct forecasts is equal to the number expected by chance. Negative skill scores are possible. E_c is defined as follows:

$$E_c = F_r(R/T) + F_{nr}(NR/T)$$

where F_r = the number of rain forecasts,

F_{nr} = the number of no rain forecasts,

R = the number of observed rain periods, and

NR = the number of observed no rain periods.

In order to test the forecasts against a skilled forecaster who has all the current data, these forecasts were compared with those the U. S.

Weather Bureau published in the Monterey Peninsula Herald. Since this paper does not publish on Sundays, no forecasts were available for Mondays. From Table 6 the verification was 86% with a skill score of 0.67. The skill score was significantly better for this subjective forecast than for the first approximation of the objective forecast.

		Forecast		
		Rain	No Rain	Total
	Rain	15	6	21
Observed	No Rain	4	48	52
	Total	19	54	73

Table 6

Data from December 1946, March 1947 and December 1948 were added to the first test data and verification for the six-month sample is shown in Table 8. The forecasts were 75% correct with a skill score of 0.29. The last three months tested were not compared with U. S. Weather Bureau forecasts.

		Forecast		
		Rain	No Rain	Total
	Rain	15	18	33
Observed	No Rain	19	99	118
	Total	34	117	151

Table 7

Although these results were somewhat discouraging, they actually are better than those obtained by using the original data. From Table 9 there was 65% verification with a skill score of 0.22.

		Forecast		
		Rain	No Rain	Total
	Rain	54	67	121
Observed	No Rain	50	161	211
	Total	104	228	332

Table 8

Table 9 shows the breakdown of the original data and the independent data for each ten-percent interval. The class mark, plotted as the expected value, is the mid-point of the interval.

	Original Data		Independent Data	
Expected	No. of Cases	% Rain	No. of Cases	% Rain
$4\frac{1}{2}$%	38	10	17	18
$14\frac{1}{2}$	43	12	7	29
$24\frac{1}{2}$	51	31	7	0
$34\frac{1}{2}$	48	31	12	25
$44\frac{1}{2}$	38	45	10	30
$54\frac{1}{2}$	40	35	5	20
$64\frac{1}{2}$	37	70	4	25
$74\frac{1}{2}$	25	52	6	50

Table 9

The failure to verify each area even for the original data must be due to the procedure of constructing the isopleths of percentage. The fact that the block diagram, Figure 7, was made from the scatter diagram, Figure 6, by constructing squares containing approximately the same number of observations automatically gives a larger weighting factor to the larger squares which have the fewer observations. Possibly the lines could be drawn more accurately on a chart which had the scales distorted so that each unit of area would have about the same number of observations.

6. Preparation of a 3-Parameter Forecast

In order to introduce new parameters not involving temperatures and heights, the first approximation to the percentage occurrence of rain was plotted separately against the wind speed and the wind direction at Oakland. The graph of wind speed, Figure 9, was abandoned with a six-month plot because the wind speed did not seem to be producing a further separation of "rain" and "no rain." The 500-mb wind direction at Oakland seemed to give more significant results. This scatter diagram, Figure 10, was analyzed into 13 blocks having 14 or more observations each. Figure 11 shows this block diagram, which was converted into the forecasting graph of Figure 12.

Of the independent data previously used, only the chronologically earlier data included 500-mb winds. This three month period forecast correctly 68% with a skill score of -0.08 using the 2-parameter forecast and 73% correct with a skill score of 0.11 for the 3-parameter forecast. The second prediction graph improved the initial forecast only slightly. With this small sample and these low skill scores one can only conclude that if it improves the forecast, it improves it only slightly. Table 10 shows the breakdown of the independent data for the 3-parameter forecast.

		Forecast		
		Rain	No Rain	Total
	Rain	4	19	23
Observed	No Rain	8	41	49
	Total	12	60	72

Table 10

It is interesting to note that the area in Figure 11 right next to the area of 84% occurrence of rain did not have a single instance of rain

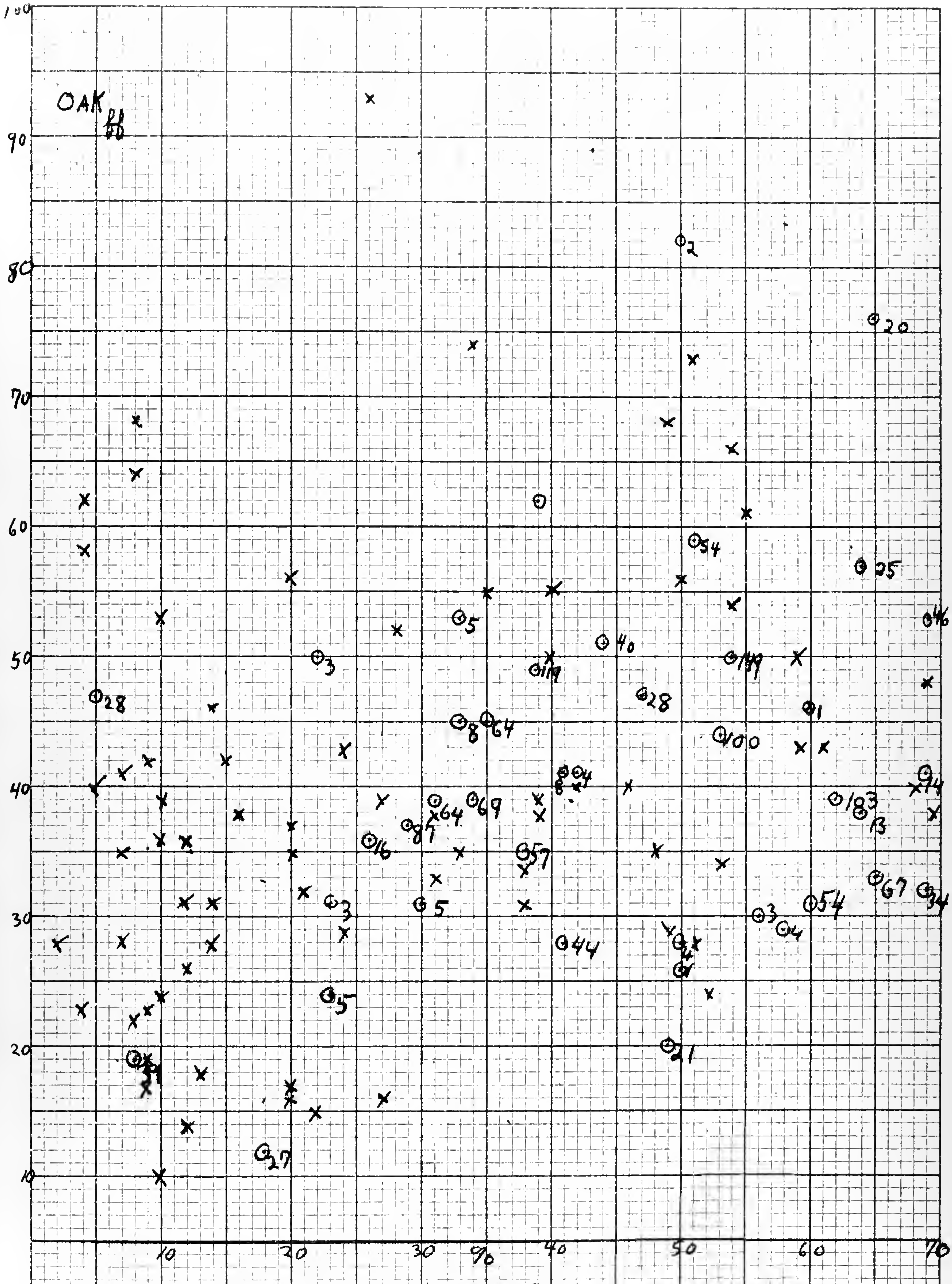


FIGURE 9.

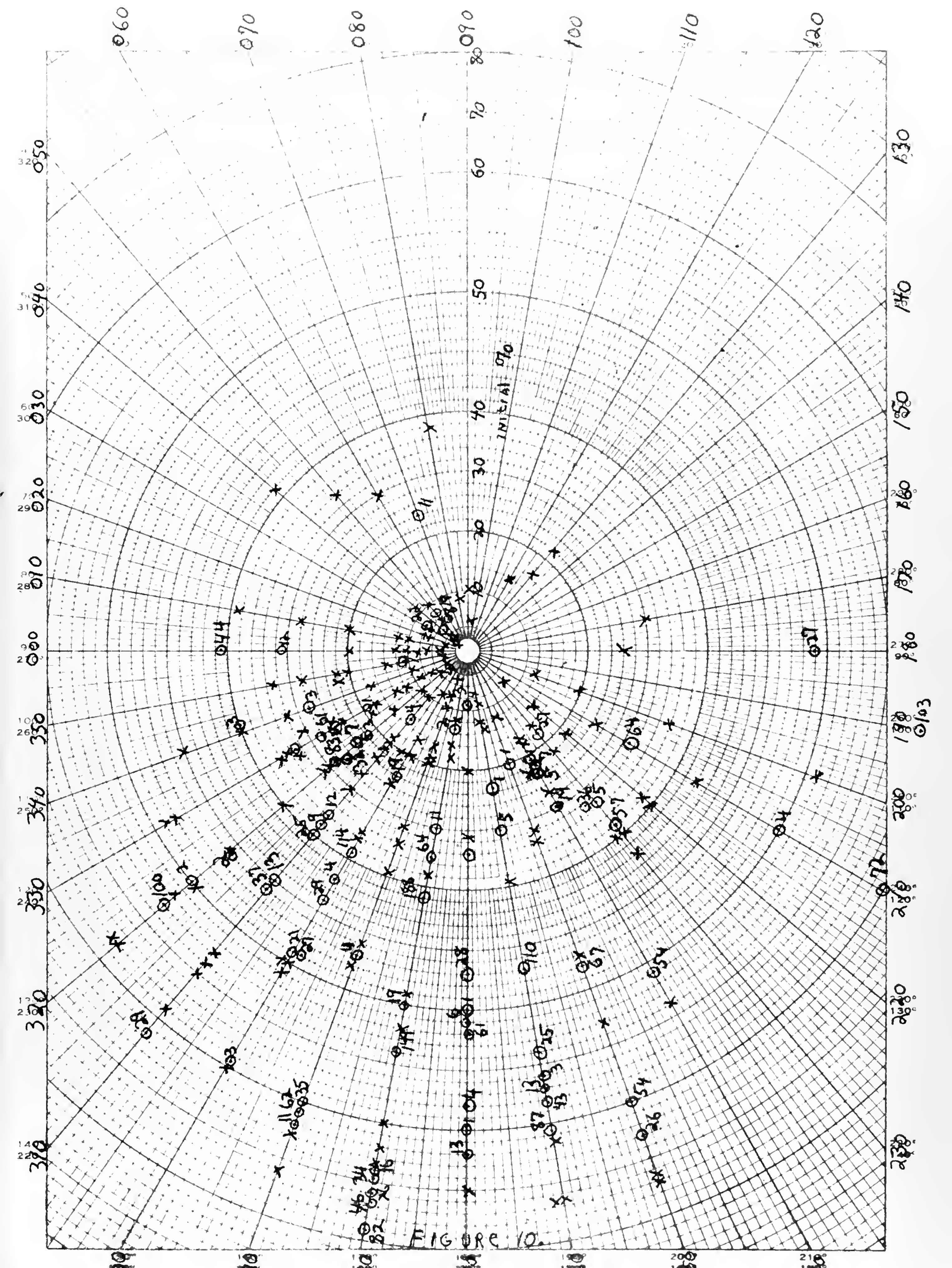


FIGURE 10.

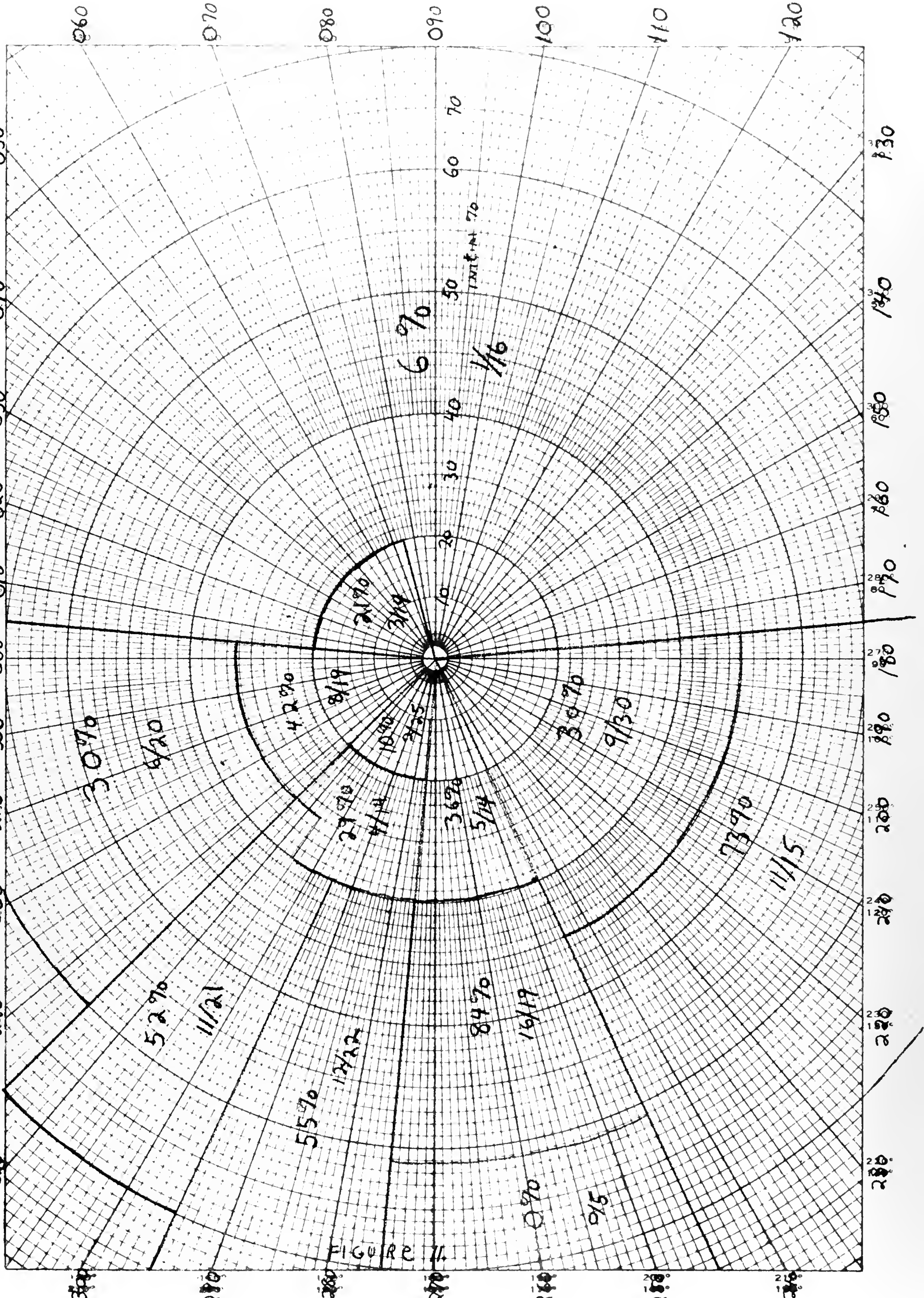


FIGURE 11.

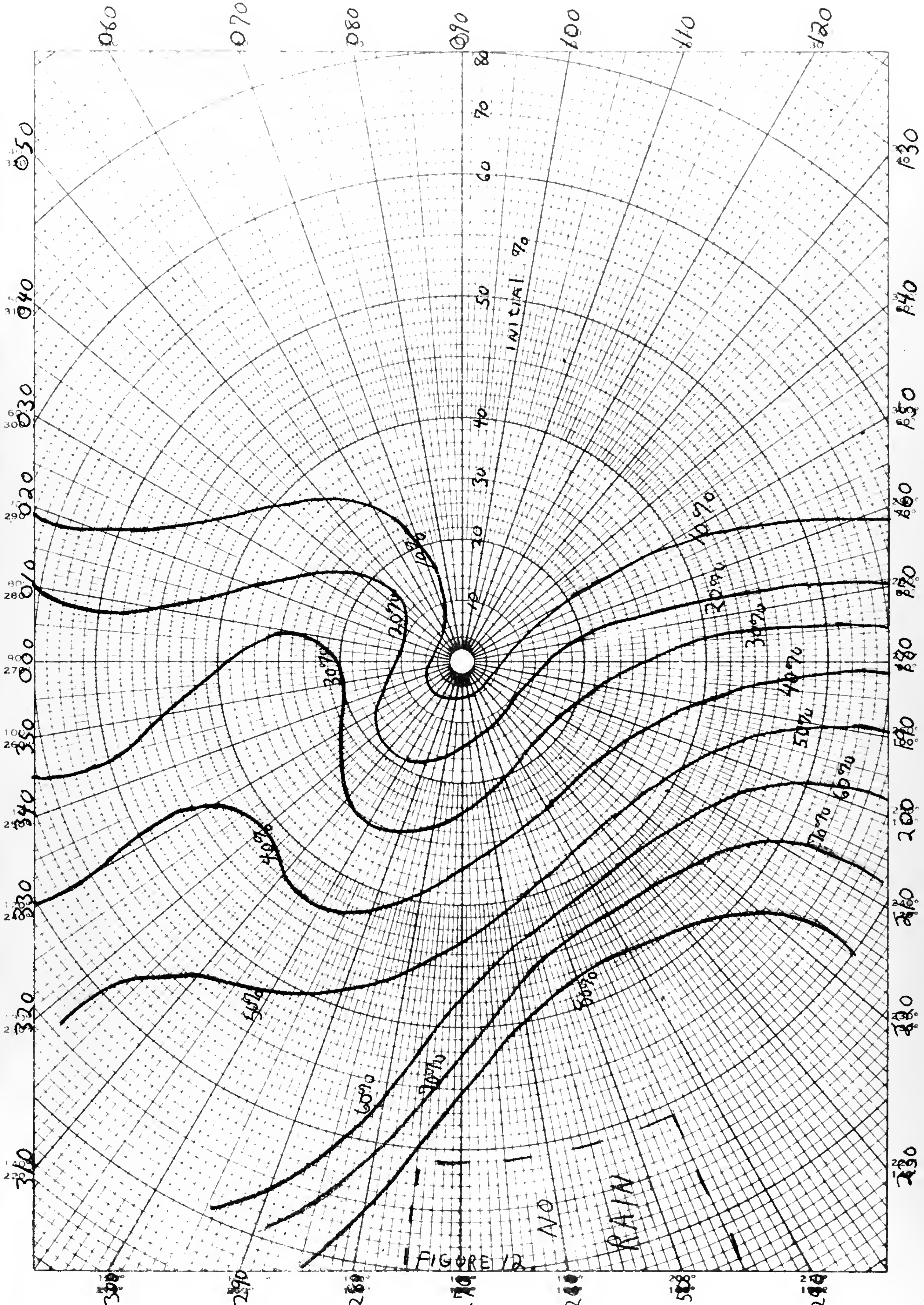


FIGURE 12.

in the eight observations both in the dependent and independent data.

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